EARLY EXERCISE OF FOREIGN CURRENCY OPTIONS:

DETERMINANTS OF AMERICAN PREMIUM AND THE CRITICAL EXCHANGE RATE

Frank J. Fabozzi,* Shmuel Hauser,† and Uzi Yaari‡

I. INTRODUCTION

Roll [11] demonstrates that the probability of early exercise of equity call options is low for small dividend payouts. Geske and Shastri [9] show that unless dividends are small, put equity options would not be exercised early. Subsequently, Shastri and Tandon [12] argue that the probability of early exercise in foreign currency options is small since foreign interest rates are analogous to a continuous dividend payout. Based on this observation, they conclude that a European model is well-suited for pricing American

*Sloan School of Management, MIT.
†Israel Securities Authority and Hebrew University.
‡School of Business, Rutgers University–Camden.

Advances in Futures and Options Research,
Volume 4, pages 219–236.

ISBN: 1-55938-060-8

foreign currency options, unless the foreign interest rate is unusually high/low for call/put options. In further justification they argue that pricing errors of a European option model are insignificant.[1]

This paper compares the Barone-Adesi–Whaley [BA-W; 2] American option-pricing model with the Garman–Kohlhagen–Grabbe [G-K-G; 6, 10] European model and tests the conditions under which foreign exchange options convey opportunities to profit from premature exercise. Our results indicate the following:

(1) The BA-W model is only advantageous in pricing out-of-the-money long-term options.
(2) The probability of gainful early exercise in puts is more sensitive to the interest rate differential, time to maturity, and volatility than that of calls.
(3) The critical spot rate in the BA-W model is based on the probability of gainful early exercise on a given date, not after that date. Based on this criterion, we find a large number of opportunities for early exercise among in-the-money options maturing in less than 45 days.

The following section presents the competing models for pricing currency options and identifying the critical spot rate, namely, the foreign currency price at or above/below which calls/puts would be exercised early. Section III examines the sensitivity of an early-exercise strategy to the option parameters of time-to-maturity, volatility, strike price, and interest rates. Section IV offers empirical findings on the BA-W model and the opportunities to gain from early exercise. Concluding remarks are presented in Section V.

II. THE MODEL

Options on foreign currency traded on the Philadelphia Exchange are contracts conveying the right, but not the obligation, to buy or sell a foreign currency at a fixed price until a fixed date. Grabbe [10] shows that the value of a European call option on a foreign currency can be obtained from[2]

$$c = SB^*N(d_1) - XBN(d_2), \tag{1}$$

and the value of a European put option from

$$p = -SB^*N(-d_1) + XBN(-d_2), \tag{2}$$

where

$$d_1 = [\ln(S/X) - \ln B^* + \ln B + 0.5\sigma^2 T]/\sigma\sqrt{T};$$
$$d_2 = d_1 - \sigma\sqrt{T};$$
$$\sigma = \int_0^T \frac{1}{T}[\sigma_G^2(u) + \sigma_B^2(u) - 2\sigma_{GB}(u)]\,du;$$

and

$N(\cdot)$ = cumulative univariate normal distribution;
G = SB^*;
S = spot price of a foreign currency;
B = domestic currency price of a domestic pure discount bond;
B^* = foreign currency price of a foreign pure discount bond;[3]
σ_S = standard deviation of percentage changes in S;
σ_B = standard deviation of percentage changes in B;
σ_{B^*} = standard deviation of percentage changes in B^*;
σ_{GB} = covariance of percentage changes in B and SB^*;
T = time to maturity, in years.

Unlike a European option, an American option can be exercised prior to expiration. In foreign currency options, this is a valuable feature since the holder has the opportunity either to exercise the call/put option, earning the foreign/domestic interest rate, or to hold the call/put to maturity, earning the domestic/foreign interest rate. Barone-Adesi and Whaley [2] offer a method for estimating the value of the early exercise privilege.[4] Given the choice of immediate exercise, the value of an American call option on foreign currency is approximated by

$$C = \begin{cases} c + Ka_2 S^{Q_2}, & \text{if } S < S^c, \\ S - X, & \text{if } S > S^c \end{cases} \quad (3)$$

and that of an American put option by

$$P = \begin{cases} p + Ka_1 S^{Q_1}, & \text{if } S > S^c \\ X - S, & \text{if } S \leqslant S^c \end{cases} \quad (4)$$

where

$$K = 1 - e^{-rT};$$

$$a_1 = -\{1 - e^{-r^*T}N[d_1(S^c)]\}/KQ_1(S^c)^{Q_1-1};$$
$$a_2 = \{1 - e^{-r^*T}N[d_1(S^c)]\}/KQ_2(S^c)^{Q_2-1};$$
$$Q_1 = \frac{1}{2}\{1 - N + \frac{1}{2}[(N - 1)^2 + 4M/K]\};$$
$$Q_2 = \frac{1}{2}\{1 - N + \frac{1}{2}[(N - 1)^2 + 4M/K]\};$$
$$M = 2r/\sigma^2 \text{ and } N = 2(r - r^*)/\sigma^2;$$

S^c is the current critical spot price of foreign currency, and r and r^* are the domestic and foreign interest rates, respectively.

Note that the critical spot rate is the exchange rate at which the investor is indifferent about exercising the option immediately. At an exchange rate higher/lower than the critical rate, the call/put option will be exercised immediately.

III. OPTIMALITY OF EARLY EXERCISE: SENSITVITY ANALYSIS

At each instant of time, the holder of an American foreign currency call/put option is faced with the decision to exercise the option and earn the foreign/domestic interest rate or continue to hold the option and earn the domestic/foreign interest rate [13]. The higher/lower the foreign interest rate, the lower/higher is the value of the call/put option and the higher/lower is the probability of gainful early exercise.

In this section, sensitvity analysis is carried out to examine the conditions for gainful early exercise. The BA-W model is utilized to identify critical spot rates, which are used in turn to compute the currency option values and the premium of American options vis-à-vis European options. Critical spot rates are numerically calculated for calls and puts based on Eqs. (3) and (4), respectively [15]. The basic set of parameters of both calls and puts is $S = 100$, $X = 100$, $\sigma_S = 0.10$, $r = 0.10$ and $T = 0.10$, where the last parameter reflects our expectation for the relevant time of optimal early exercise. Our expectation for early exercise only in the short maturity is based on our simulations and trading data. The parameters used are changed one at a time to trace effects of the three factors under study: cross-country rate differential, time-to-maturity, and exchange rate volatility. In each case, the effect is defined (1) by the percentage of American premium as measured by $(C\text{-}c)/c$ for calls and by $(P\text{-}p)/p$ for puts, or (2) by the critical spot rate, S^c

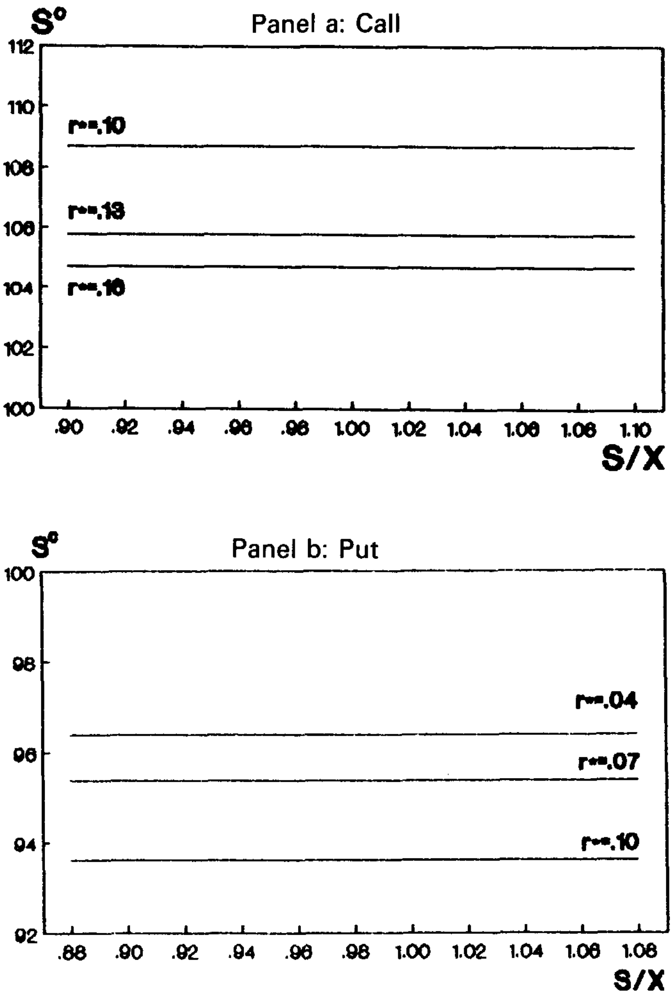

Figure 1. Effects of the degree in- or out-of-the money on the critical spot rate (X = 100, Sig = .1, T = .1, r = .1)

Effects on call options are displayed in panel a of Figures 1–6 and those on puts in panel b of the same figures.

Figures 1a and 1b illustrate the effects of the degree in-the-money/out-of-the-money, S/X, on the critical spot rate S^c for calls and puts, respectively. Two features are worth noting. First, in both calls and puts the critical spot rate remains unaffected by the extent to which options are in-the-money/out-of-the-money. Since X is held constant, this implies that S^C is not affected by S. In the

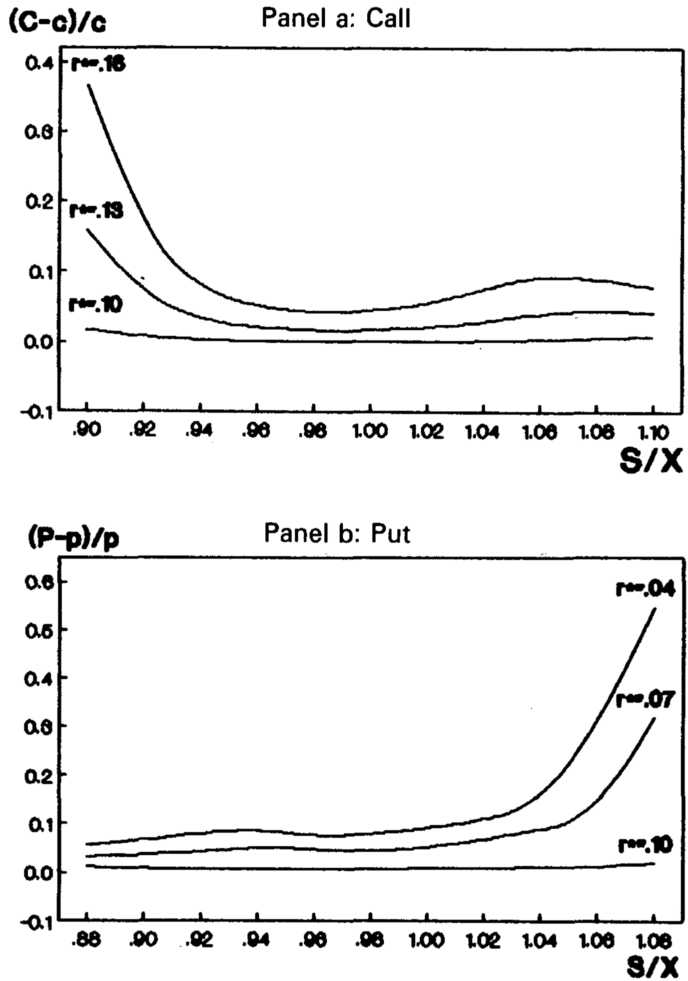

Figure 2. Effects of the degree in- or out-of-the money on the relative American premium (X = 100, Sig = .1, T = .1, r = .1)

analysis of other effects below, this feature allows us to use at-the-money options without loss of generality. Second, with a constant domestic interest rate *r*, an increase/decrease in the foreign rate differential $r^* - r$ causes a decrease/increase in the critical spot rate for calls/puts. Respectively, this implies that the opportunity for early exercise in calls/puts increases with an increase/decrease in the foreign interest rate r^*. This feature holds in parallel with the familiar negative/positive effect of r^* on call/put prices.

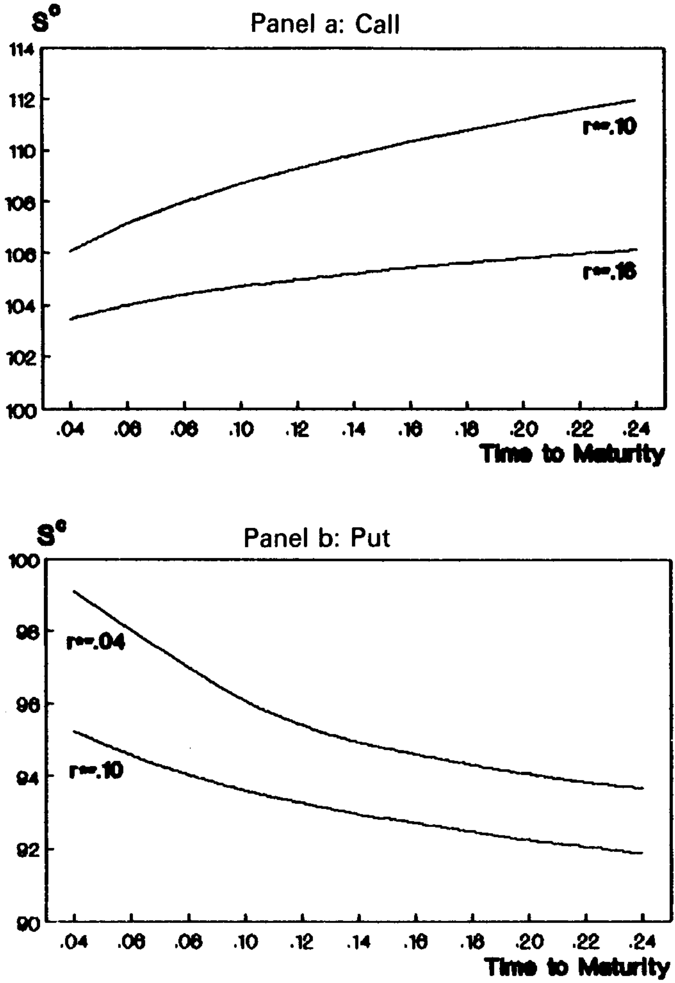

Figure 3. Effects of time to maturity on the critical spot rate (S = 100, X = 100, r = .1, Sig = .1)

Other effects of S/X on the relative American premium of calls, $(C\text{-}c)/c$, and puts, $(P\text{-}p)/p$, are illustrated in Figures 2a and 2b, respectively. Overall, both figures show that the deeper options are out-of-the-money, the higher is the relative American premium. This occurs despite the fact that option prices increase with S/X. Another feature seen from the graphs is that the American premium is increasing with an increase in the absolute rate differential $|r^* - r|$, where the foreign interest rate is above/below the

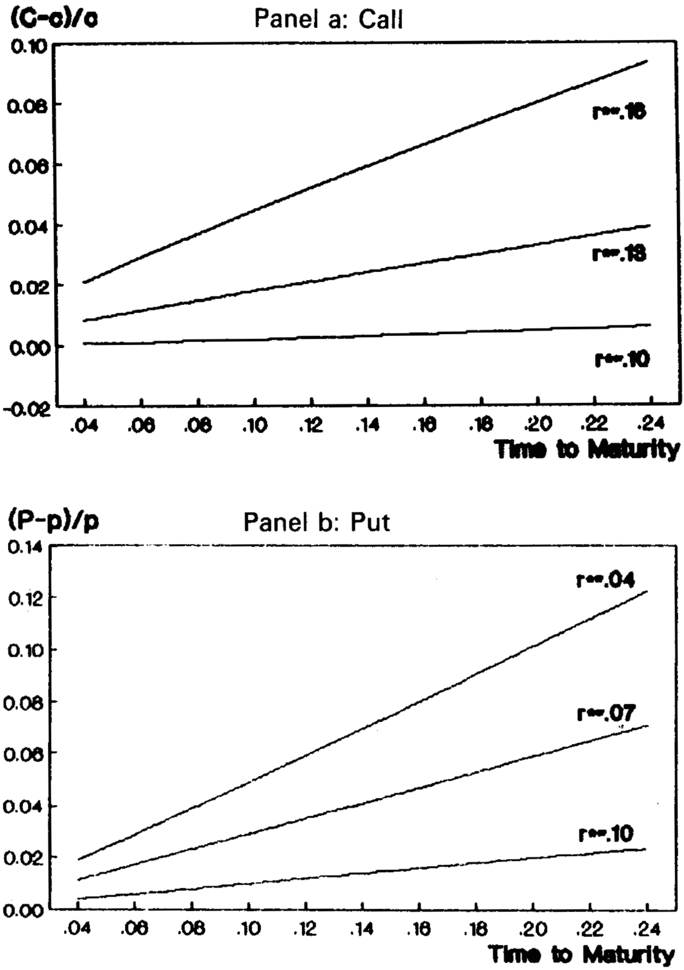

Figure 4. Effects of time to maturity on the relative American premium (S = 100, X = 100, r = .1, Sig = .1)

domestic rate for calls/puts. When this relationship between the two rates is reversed, the American premium vanishes. The last scenario aside, the simulations presented indicate that the relative American premium in out-of-the-money puts is more sensitive to the interest rate differential than that of out-of-the-money calls.

A closer look at these figures reveals a nonmonotonic behavior of the functions displayed, a feature reflecting the fact that American options are more sensitive than European ones to

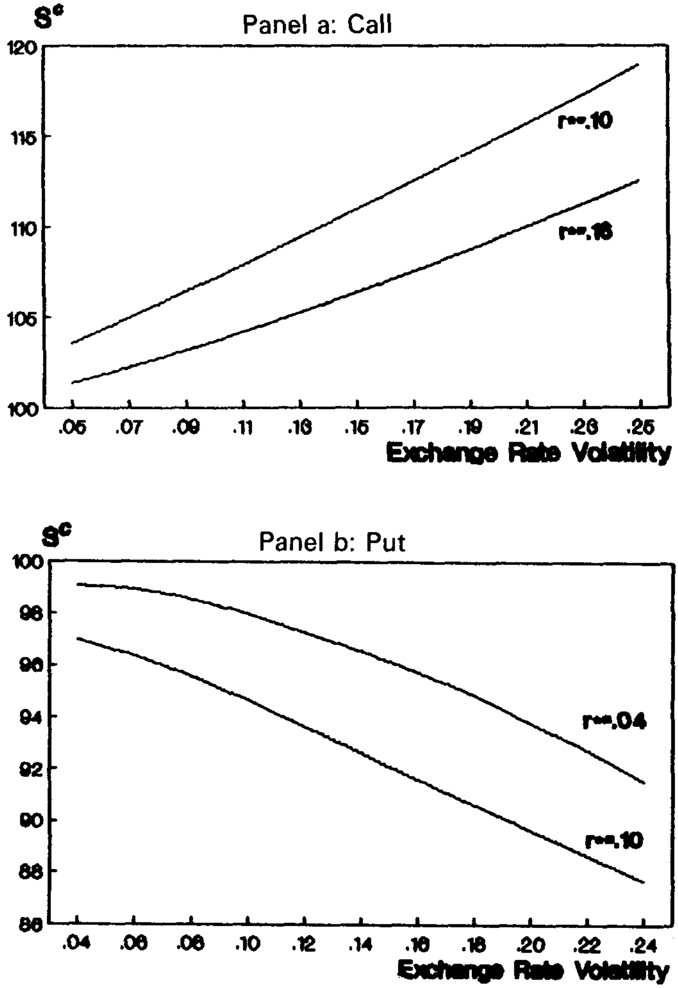

Figure 5. Effects of exchange rate volatility on the critical spot rate (S = 100, X = 100, r = .1, T = .1)

changes in the time value of money set by the foreign rate differential. Moreover, since the price of out-of-the-money options is fully determined by the time value of money, it is not surprising to find that the effect of the foreign rate differential is more pronounced in those options. Following this logic, the first change in the slope from negative to positive results from the diminishing time value effect increasingly dominated by the intrinsic value effect, a change caused by moving from an out-of-the-money to in-the-money

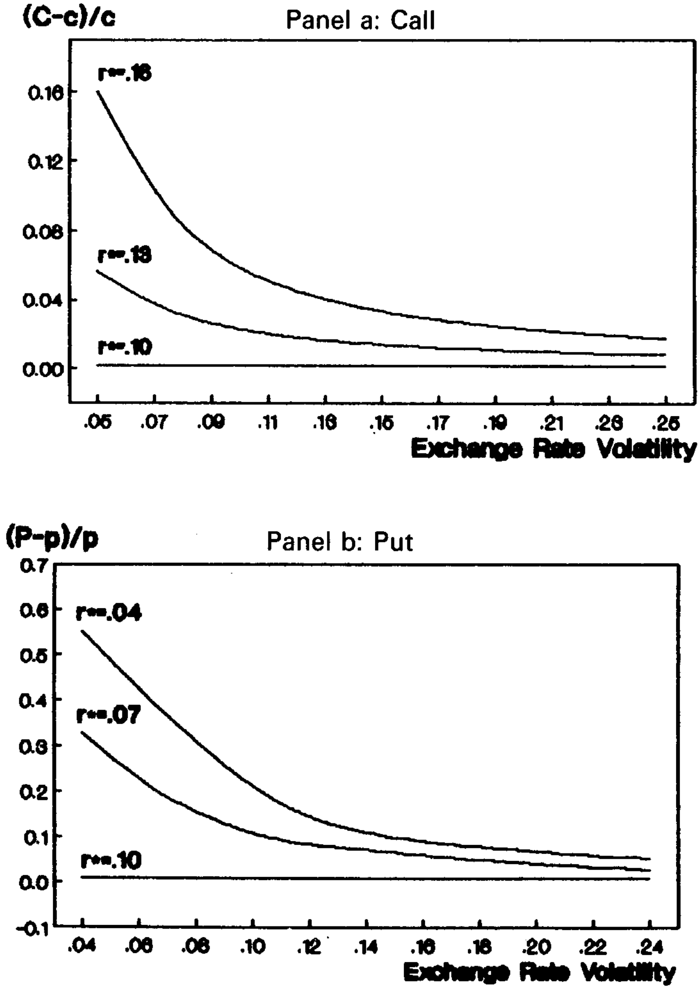

Figure 6. Effects of exchange rate volatility on the relative American premium (S = 100, X = 100, r = .1, T = .1)

scenario. The second change in the sign of the slope, when present, reflects once again the fact that the time value effect dominates that of the intrinsic value ($S - X$)—a development occurring when the option should be exercised early according to the model.

An important practical implication of the simulation results presented thus far is that use of the American option model is advantageous in pricing out-of-the-money as well as in-the-money options. This is because empirical studies show that theoretical pricing of out-of-the-money options is downward-biased relative to actual prices.

Effects of the time-to-maturity T on the critical spot rate S^c in calls and puts are displayed in Figures 3a and 3b, respectively. The salient feature here is a critical spot rate that is monotonically increasing for calls and decreasing for puts. In both cases the implication is that the opportunity to gain from early exercise is inversely related to the time to maturity. As illustrated by Figures 4a and 4b, this relationship holds despite the fact that the relative American premium diminishes as the option approaches maturity. This peculiarity is explained by the fact that the shorter time to maturity decreases the probability that current spot rates will critically change relative to S^c.

Effects of exchange rate volatility σ on the critical spot rates of calls and puts are illustrated in Figures 5a and 5b, respectively. The critical spot rate is related to volatility directly for calls and inversely for puts. In both cases, a lower volatility means a greater opportunity for gainful early exercise. This statement is consistent with Figures 6a and 6b, which show that for both calls and puts the lower the volatility, the higher is the relative American premium. Here again, a lower volatility of exchange rates decreases the probability that current spot rates will critically change relative to S^C. Unnoticed by previous writers, these graphs further indicate that the relative American premium is more sensitive to changes in volatility when the foreign rate differential is higher, and more so for puts than for calls. The difference in sensitivity between puts and calls, as measured by a change in the relative American premium for a given change in volatility, is apparent in the lower range of volatilities.

IV. EMPIRICAL FINDINGS

The above simulation first demonstrated the known result that in-the-money calls/puts may be optimally exercised prior to maturity when the foreign interest rate is higher/lower than the domestic one. It was then shown that the closer the expiration date and the lower the volatility, the higher is the likelihood of early exercise of deep-in-the-money options. It was further shown that the early-exercise decision for puts is more sensitive to changes in the underlying parameters than for calls. In this section, we test for the presence of opportunities of early exercise by identifying options that should be exercised early according to the BA-W model. This is accomplished after testing for the ability of the BA-W model vs. that of G-K-G to predict currency option prices.

The data used are observations on options of four foreign currencies traded on and compiled by the Philadelphia Exchange from August 1983 to December 1984. Eurocurrency interest rates are reported by the *Financial Times*. Volatility on each day is estimated by Whaley's [14] procedure and used as an input in the following trading day. The four currencies included are the British pound (BP), West German mark (DM), Japanese Yen (JY), and Swiss franc (SF). In the sample period, U.S. interest rates were lower than British interest rates and higher than those in Japan, Germany, and Switzerland.

A. Pricing by the BA-W and G-K-G models

The first objective is to compare the performance of the European and American models in pricing currency options. To this end, the relative mean-squared errors (RMSE) are computed by

$$\begin{aligned} \text{RMSE} &= \text{MSE}_{\text{GKG}}/\text{MSE}_{\text{BAW}} \\ &= \frac{1}{n}\sum_{i=1}^{n}(C_{\text{GKG}} - C_{\text{MKT}})^2 \Big/ \frac{1}{n}\sum_{i=1}^{n}(C_{\text{BAW}} - C_{\text{MKT}})^2, \end{aligned}$$

where MSE is the mean squared error, C_{GKG} and C_{BAW} are option prices according to the G-K-G and BA-W models, respectively, and C_{MKT} is the observed option market price. According to Ashley et al. [1], the significance of this ratio is determined by calculating the following regression:

$$\Omega = a + b(\Phi - \mu) + \varepsilon$$

where

$$\Omega = e_1 - e_2, \quad \Phi = e_1 + e_2, \quad e_1 = C_{\text{MKT}} - C_{\text{GKG}}, \quad e_2 = C_{\text{MKT}} - C_{\text{BAW}}$$

and μ is the mean of Φ. This regression is shown to be equivalent to the hypothesis H_0: $\text{MSE}_{\text{GKG}} = \text{MSE}_{\text{BAW}}$ vs. H_1: $\text{MSE}_{\text{GKG}} >$

Table 1. Call Option Pricing by European vs. American Models: Average American Premium and RMSE by Maturity and Option Class (August 1983–December 1984)

Maturity (days)	*In*	*At*	*Out*	*Total*
British pound ($r^* > r$)				
$T \leq 45$				
n	66	150	138	354
RMSE	1.028	1.001	1.000	1.001
premium (%)	0.286	0.129	7.628	3.082
$45 < T \leq 90$				
n	48	160	280	488
RMSE	1.152	1.023	1.012	1.033
premium (%)	0.663	0.686	1.237	1.000
$90 < T$				
n	153	324	639	1116
RMSE	1.269*	1.090	1.057	1.107
premium (%)	4.700	1.891	0.923	2.015
Total				
n	267	634	1057	1958
RMSE	1.235*	1.052	1.009	1.031
premium (%)	0.700	2.660	1.882	1.955
German mark, Japanse yen, and Swiss franc ($r^* < r$)				
Total				
n	1143	2471	4611	8225
RMSE	1.037	1.003	1.001	1.005
premium (%)	0.018	0.006	0.013	0.012

*Significantly different from unity at the 5% level.

MSE_{BAW}. According to this test, if *a* and *b* are significantly negative, the G-K-G model cannot be judged as a significant improvement over the BA-W model. If one of the coefficients is negative but not significant, a one-tailed *t*-test on the other coefficient can be used. If both coefficients are positive, an *F*-test can be employed to test the hypothesis by $a = b = 0$.[5]

Tables 1 and 2 report the RMSE and average American premium by maturity and option class for calls and puts, respectively. The results reveal the following patterns. First, when the probability of premature exercise is low, there is no advantage in using the BA-W model. For the BP puts, where the foreign interest rate is higher

Table 2. Put Option Pricing by European vs. American Models: Average American Premium and RMSE by Maturity and Option Class (August 1983–December 1984)

Maturity (days)	*In*	*At*	*Out*	*Total*
German mark, Japanese yen, and Swiss franc ($r^* < r$)				
$T \leq 45$				
n	173	340	117	630
RMSE	0.756*	0.945	1.310*	0.926
premium (%)	5.235	5.320	27.691	9.453
$45 < T \leq 90$				
n	163	238	96	497
RMSE	0.716*	1.827*	1.368*	0.853*
premium (%)	10.598	13.504	20.511	13.904
$90 < T$				
n	228	418	224	870
RMSE	0.827*	3.644*	2.479*	1.383*
premium (%)	18.725	19.027	18.917	18.919
Total				
n	564	996	457	1997
RMSE	0.767*	2.057*	2.095*	1.087
premium (%)	12.238	13.028	21.618	14.685
British pound ($r^* > r$)				
Total				
n	300	414	240	954
RMSE	1.109	1.049	1.169	1.099
premium (%)	2.079	2.081	2.304	2.181

*Significantly different from unity at the 5% level.

than the domestic one, and for the DM, JY, and SF calls, where the interest rate relationship is reversed, the results indicate RMSE that are insignificantly different from unity. All these cases are characterized by a low American premium. Second, when the domestic interest rate substantially exceeds the foreign one, there is a perceptible difference in the pricing of the two models. Table 2 shows that the BA-W model is more advantageous in pricing puts the further out-of-the-money they are and the longer their time to maturity. Symmetrically, this model is inferior in pricing in-the-money puts. These results are consistent with previous empirical studies reporting systematic under-/overpricing of out-of-/in-the-money options using the European model [4, 13]. The implication is that the additional value in the American model due to the early-exercise privilege serves to decrease the bias in out-of-the-money options and increase the bias in in-the-money ones. Third, when the domestic interest rate is below the foreign rate, a scenario suggesting a potential early exercise of calls, the use of the American model is not clearly advantageous. This is evident in Table 1, where the RMSE is insignificantly different from zero in BP calls. The lack of clear advantage to the American model in our sample period is due to a small interest rate differential of less than 2%. Overall, these results indicate that the BA-W model is not uniformly superior or inferior to the G-K-G model in pricing foreign currency options.

B. Early Exercise by the BA-W Model

In our sample period, domestic interest rates were lower than those in England and higher than those in Japan, Germany, and Switzerland. Based on the analysis in Section III, early exercise should be more frequent for deep-in-the-money BP calls and for in-the-money DM, JY, and SF puts. We now seek to determine whether our sample contains opportunities for early exercise based on the BA-W model. Note that the BA-W model gives the probability of gainful early exercise on a given date, not after that date. We do not investigate the separate question of whether an early-exercise decision based on the model would turn out to be profitable.

The results displayed in Table 3 include only deep-in-the-money options, where the theory predicts opportunities to gain from early exercise. In general, the optimal time to expiration of all options in all currencies averaged less than 30 days and did not exceed 45 days. These figures are consistent with the view that opportunities to gain from early exercise are absent from in-the-money options with maturities exceeding 45 days. One feature of these results is that the average implied standard deviation of optimally exercised options is lower than that of the total sample—confirming the theoretical inverse relationship found between volatility and gainful early exercise. Another noteworthy feature is that 42% of deep-in-the-money put options (12% of the total sample), compared with only 5% of call options (less than 1% of the total sample) should have been exercised prior to maturity. All of these results confirm those obtained above by simulation, showing that the relative American premium of puts is larger and more sensitive to changes in the

Table 3. Selected Statistics on Optimally Early-Exercised Options According to the BA-W Method (August 1984–December 1985)[a,b]

	Call (S/X > 1.02)	*Put (S/X < 0.98)*		
	BP	*DM*	*JY*	*SF*
No. of options	1958	755	614	628
No. of deep in-the-money options	267	189	159	216
No. of exercised options	13	81	52	108
σ_t	0.113 (0.031)	0.110 (0.029)	0.104 (0.012)	0.106 (0.018)
σ_e	0.079 (0.036)	0.099 (0.011)	0.096 (0.005)	0.101 (0.008)
r	9.341 (0.685)	10.799 (0.861)	11.392 (0.863)	11.144 (0.861)
r^*	9.997 (0.529)	5.713 (0.377)	6.155 (0.313)	3.054 (0.501)
T	0.136 (0.051)	0.071 (0.034)	0.102 (0.063)	0.049 (0.024)
premium (%)	3.071 (2.81)	11.11 (12.4)	7.391 (6.82)	24.955 (23.1)

[a]Numbers in parentheses are standard errors.
[b]T is the average time to maturity of optimally exercised options; r, r^* the average interest rates for the total sample of exercised options; σ_t, σ_e the average implied standard deviation of the total sample and of optimally exercised options, respectively; and premium is the average American premium.

underlying parameters than that of calls. Obviously, the observed high percentage of early-exercised puts is also attributed to the fact that the absolute difference between domestic and foreign interest rates happened to be greater in the currencies used to test puts than in those used to test calls.

IV. CONCLUDING REMARKS

This paper compared the performance of the BA-W American model and G-K-G European model in pricing American currency options. It was shown that the BA-W model is superior to the

G-K-G model in pricing out-of-the-money long-term put options and inferior in pricing in-the-money short-term put options. The two models performed equally well in pricing call options. It was further shown that the interest rate differential across countries has a greater effect on the probability of gainful early exercise in foreign currency puts than that of calls. The American model identified a large number of opportunities in our sample for gainful early exercise among in-the-money options maturing in less than 45 days. These findings suggest the need for a further investigation of the ex-post consequences of early exercise decisions based on the American model. In combination with our results, such an investigation would be useful in the developments of trading strategies and the testing of market efficiency.

NOTES

1. In fact, Shastri and Tandon [12] find insignificant pricing errors in using the European option model to price American calls on foreign currency, but significant errors in pricing American puts by the European option model.
2. Biger and Hull [3] and Garman and Kohlhagen [6] develop similar models on the assumption of deterministic interest rates. Their models conform to Grabbe's when the generating processes of the pure domestic and foreign bond prices are Wiener processes with a zero standard deviation of bond prices.
3. Given the domestic yield r or the foreign yield r^*, the prices of domestic and foreign pure discount bonds are determined by $B = e^{-rT}$ and $B^* = e^{-r^*T}$, respectively.
4. This method can be generalized to any commodity option by replacing the foreign interest rate r^* by $r - b$, where r is the domestic interest rate and b the cost of carrying the commodity. See, for example, Whaley [15] on options on futures contracts.
5. If RMSE < 1, the alternative hypothesis is H_1: $MSE_{BAW} > MSE_{GKG}$.

REFERENCES

1. Ashley, R., Granger, C. W. and Schmalensee, R. "Advertising Aggregate Consumption: An Analysis of Causality," *Econometrica* 48 (1980) 1149–1167.
2. Barone-Adesi, G. and Whaley, R. E. "Efficient Analytic Approximation of American Option Values," *Journal of Finance* 42 (1987) 301–320.
3. Biger, N. and Hull, J. "The Valuation of Currency Options," *Financial Management* 12 (1983) 24–28.
4. Bodurtha, J. N. and Courtadon, G. R. "Efficiency Tests of the Foreign Currency Options Market," *Journal of Finance* 41 (1986) 151–162.
5. Bodurtha, J. N. and Courtadon, G. R. "Tests of an American Option Pricing Model on the Foreign Currency Options Market," *Journal of Financial and Quantitative Analysis* 22 (1987) 153–167.

6. Garman, M. B. and Kohlhagen, W. S. "Foreign Currency Option Values," *Journal of International Money and Finance* 2 (1983) 231–237.
7. Geske, R. and Johnson, H. "The American Put Option Valued Analytically," *Journal of Finance* 39 (1984) 1511–1524.
8. Geske, R. and Roll, R. "On Valuing American Call Options with the Black–Scholes European Formula," *Journal of Finance* 39 (1984) 443–455.
9. Geske, R. and Shastri, K. "The Early Exercise of American Puts," *Journal of Banking and Finance* 9 (1985) 207–219.
10. Grabbe, J. O. "The Pricing of Call and Put Options on Foreign Exchange," *Journal of International Money and Finance* 2 (1983) 239–254.
11. Roll, R. "An Analytical Valuation Formula for Unprotected American Call Options on Stock with Known Dividends," *Journal of Financial Economics* 5 (1977) 251–258.
12. Shastri, K. and Tandon, K. "On the Use of European Models to Price American Options on Foreign Currency," *Journal of Futures Markets* 6 (1986) 93–108.
13. Shastri, K. and Tandon, K. "Valuation of Foreign Currency Options: Some Empirical Tests," *Journal of Financial and Quantitative Analysis* 21 (1986) 145–160.
14. Whaley, R. E. "Valuation of American Call Option on Dividend-Paying Stocks: Empirical Tests, "*Journal of Financial Economics*, 10 (1982) 29–58.
15. Whaley, R. E. "Valuation of American Futures Options: Theory and Empirical Results," *Journal of Finance* 41 (1986) 127–150.